

# LOSS OF MASS AND STABILITY OF GALAXIES IN MOND

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*Draft version February 1, 2008*

## ABSTRACT

The self-binding energy and stability of a galaxy in MOND-based gravity are curiously decreasing functions of its center of mass acceleration (of the order of  $10^{-12}$  -  $10^{-10}$  m/s<sup>2</sup>) towards neighbouring mass concentrations. A tentative indication of this breaking of the Strong Equivalence Principle in field galaxies is the RAVE-observed escape speed in the Milky Way. Another consequence is that satellites of field galaxies will move on nearly Keplerian orbits at large radii (100 - 500 kpc), with a declining speed below the asymptotically constant naive MOND prediction. But consequences of an environment-sensitive gravity are even more severe in clusters, where member galaxies accelerate fast: no more Dark-Halo-like potential is present to support galaxies, meaning that extended axisymmetric disks of gas and stars are likely unstable. These predicted reappearance of asymptotic Keplerian velocity curves and disappearance of “stereotypic galaxies” in clusters are falsifiable with targeted surveys.

*Subject headings:* gravitation - dark matter - galaxies: structure - galaxies: kinematics and dynamics

## 1. INTRODUCTION

The exact nature of dark matter is an outstanding puzzle despite our ability to carry out increasingly realistic simulations. No astroparticle or gravitational theory so far can account for its various effects on both galactic and large scales satisfactorily (e.g., Zhao 2007).

In relatively isolated (field) galaxies, observations of a tight correlation between the mass profiles of baryonic matter and dark matter at all radii (McGaugh et al. 2007; Famaey et al. 2007a) are most often interpreted as supporting the modified Newtonian dynamics (MOND; Milgrom 1983; Bekenstein & Milgrom 1984). Indeed, without resorting to galactic dark matter, this simple prescription reproduces (to amazing consistency) the kinematics of galaxies over five decades in mass (e.g. Sanders & McGaugh 2002; Bekenstein 2006; Milgrom & Sanders 2003; Sanders & Noordermeer 2007; Nipoti et al. 2007; Famaey, Bruneton & Zhao 2007b; Gentile et al. 2007ab). The recently devised relativistic counterparts of MOND (Bekenstein 2004; Zlosnik, Ferreira & Stark-

man 2007) also enabled to study the Cosmic Microwave Background (Skordis et al. 2006) as well as gravitational lensing of galaxies and galaxy clusters (e.g., Zhao et al. 2006; Chen & Zhao 2006; Angus et al. 2007; Famaey et al. 2007c). However, it is also important to observationally and theoretically examine how internal properties of galaxies like their rotation curve and morphology should change with the environment.

Indeed MOND-based theories generate different degrees of dark matter-like effects depending on the absolute acceleration. Most galaxies are, like the Milky Way, in the field where they accelerate slowly with respect to the Cosmic Microwave Background, typically at a rate of  $0.01a_0$  to  $0.03a_0$  (Famaey et al. 2007b; Milgrom 2002; Angus & McGaugh 2007). But in X-ray clusters, galaxies accelerate much faster, from  $0.3a_0$  to  $3a_0$  (Angus et al. 2007; Pointecouteau, Arnaud & Pratt 2005).

This external gravitational field has wider and more subtle implications for the internal system in MOND than in Newton-Einstein gravity, for the very reason that MOND breaks the Strong Equivalence Principle. In particular, it is well known that MOND potentials are logarithmic for isolated distributions of finite mass, and con-

sequently infinitely deep, but that the internal potential becomes “polarised Keplerian” at large distances (Bekenstein & Milgrom 1984; Zhao & Tian 2005) when an external field is applied.

In this contribution, we *numerically* solve the MOND Poisson equation for systems embedded in several different environments, ranging from the field to galaxy clusters, and show (i) that for the Milky way (embedded in a weak gravitational field), the local escape speed is numerically compatible with the observations as analytically predicted in Famaey et al. (2007b), (ii) that rotation curves of Milky Way-like galaxies would have a rapid Keplerian fall-off when residing close to the center of clusters, while this fall-off in field galaxies would happen at 100 - 500 kpc, and (iii) that usual Low Surface Brightness disks should not exist in MONDian clusters.

## 2. BINDING ENERGY OF AN ACCELERATING MILKY WAY

Galaxies free-fall, but with slowly-changing systematic (center-of-mass) velocity  $\mathbf{v}_{com}(t)$ . Their present non-zero systematic velocity is mainly the accumulation of the acceleration by the gravity from neighbouring galaxies over a Hubble time. Consider, as a first approximation, that a galaxy is stationary in a non-inertial frame (in the Galilean sense), which free-falls with a “uniform” systematic acceleration  $\dot{\mathbf{v}}_{com} = g_{ext}\hat{X} = cst$  due to an external linear potential, say,  $-g_{ext}X$  along the  $X$ -direction, where the over-dot means time-derivatives. Let  $\dot{\mathbf{v}}_{int} = (\ddot{X}, \ddot{Y}, \ddot{Z})$  be the peculiar acceleration of a star-like test particle in the coordinates relative to the center of a non-evolving galaxy internal mass density  $\rho(X, Y, Z)$ , then

$$\dot{\mathbf{v}}_{int} = \mathbf{g} - \dot{\mathbf{v}}_{com} = -\nabla\Phi_{int}(X, Y, Z), \quad (1)$$

where  $\mathbf{g}$  is the absolute acceleration  $\mathbf{g}$  satisfying the MOND Poisson’s equation

$$-\nabla \cdot [\mu(x)\mathbf{g}] = 4\pi G\rho(X, Y, Z), \quad x \equiv \frac{|\mathbf{g}|}{a_0}. \quad (2)$$

One can define an “effective potential”  $\Phi_{int}(X, Y, Z)$  (called “internal” potential) and an “effective energy”  $E_{eff} = \frac{v_{int}^2}{2} + \Phi_{int}(X, Y, Z)$ , where  $E_{eff}$  is conserved along the orbit of the test particle effectively moving in a force field  $-\nabla\Phi_{int}(X, Y, Z)$ , which is curl-free and time-independent because the absolute gravity  $\mathbf{g}$  is curl-free, center-of-mass acceleration  $\dot{\mathbf{v}}_{com}$  is assumed a constant, and the galaxy density  $\rho(X, Y, Z)$  is assumed time-independent.

Far away from the center of the free-falling system, we have  $|\dot{\mathbf{v}}_{int}| \ll |\dot{\mathbf{v}}_{com}|$ , hence  $\mu \rightarrow \mu_m \equiv \mu(\dot{\mathbf{v}}_{com}/a_0) = cst$ , and the equation reads (Bekenstein & Milgrom 1984; Milgrom 1986; Zhao & Tian 2005; Zhao & Famaey 2006):

$$\nabla^2\Phi_{int} + \Delta \frac{\partial^2}{\partial X^2}\Phi_{int} \rightarrow 4\pi G\rho/\mu_m, \quad (3)$$

where  $Y, Z$  denote the directions perpendicular to the external field  $X$ -direction, and  $\Delta = [\ln\mu/\ln x]_{x=|\dot{\mathbf{v}}_{com}|/a_0}$  is a dilation factor (note that  $1 \leq 1 + \Delta \leq 2$ ). So at large radii where the external field dominates, and the equation is linearizable, the potential satisfies a mildly anisotropic Poisson equation, and the solution at large

radii <sup>1</sup> goes to

$$\Phi_{int}^\infty(X, Y, Z) = -\frac{GM_{int}}{\mu_m \sqrt{(1 + \Delta)(Y^2 + Z^2) + X^2 + s^2}}, \quad (4)$$

where we included a softening radius  $s$ , comparable of the half-light radius of a galaxy. Hence the internal potential  $\Phi_{int}$  is finite, and approaches zero at large radii.

The escape speed at any location  $\mathbf{r}$  in the system can then be meaningfully defined by

$$0 = E_{eff} = \frac{v_{esc}^2(X, Y, Z)}{2} + \Phi_{int}(X, Y, Z). \quad (5)$$

Such escape speed is a scalar independent of “the path to escape” because the “effective energy”  $E_{eff}$  is conserved, and a particle with  $E_{eff}$  equal zero (the maximum value of  $\Phi_{int}$ ) will reach infinite distance from the system, and never return, hence will be lost into the MOND potential of the background (from which it cannot escape). However, equal escape speed contours across a disk galaxy are generally not axisymmetric, meaning the escape speed on opposite symmetric locations of the Galaxy differ.

Hereafter, we *numerically* solve Eq. 2 using the MOND Poisson solver developed by the Bologna group (Ciotti, Londrillo, & Nipoti 2006); the results based on spherical grids are also confirmed with the cartesian grid-based code of the Paris group (Tiret & Combes 2007) with very different spatial resolutions. We program in the mass density of the internal system, solving the MOND Poisson equation as if it were isolated, except for requiring a boundary condition on the total gravity as  $-\mathbf{g} \rightarrow g_{ext}\hat{X} - \nabla\Phi_{int}^\infty(X, Y, Z)$  on the last grid point  $(X, Y, Z)$ . Note finally that in our models hereafter, we use the parametric  $\mu$ -function  $\mu(x) = x/(1 + x)$ , which fits well the rotation curve of the Milky Way (Famaey & Binney 2005), as well as external galaxies (Famaey et al. 2007a; Sanders & Noordermeer 2007).

We use the Besançon Milky Way Model (Robin et. al 2003) to simulate High Surface Brightness (HSB) galaxies. This model is a realistic representation of the Galaxy, explaining currently available observations of different types (photometry, astrometry, spectroscopy) at different wavelengths. The stellar populations included in the model are : a thin disk made of seven isothermal layers each having a different age, between 0.1 and 10 Gyr; a 11-Gyr-old thick disk with a modified exponential density law, a spheroid with a power law density, slightly flattened, a prolate old bulge modeled by a tri-axial density law. We logically take the dark matter halo away from our simulations. We then apply the MOND Poisson solver using  $512 \times 64 \times 128$  grid points where the grid points in the radial direction are chosen as  $r_i = 50.0 \tan \left[ (i + 0.5) \frac{0.5\pi}{512+1} \right] \text{ kpc}$ .

As a first application, the RAVE solar neighbourhood escape speed  $544_{-46}^{+64}$  km/s is well-reproduced by our fully numerical model galaxy (Fig. 1 for a typical external field of  $0.01a_0$ ), as analytically anticipated in Famaey et al. (2007b). When the direction of the external gravity changes, the escape speed changes in a narrow range

<sup>1</sup> throughout the paper, by “large radii” we mean a distance large enough to neglect the internal field  $g_{int} \ll \dot{v}_{com} = g_{ext}$  but small enough that the external field  $g_{ext}$  can be treated as constant

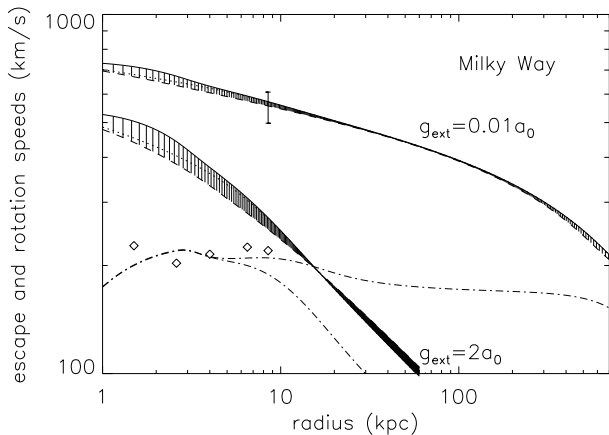


FIG. 1.— Model of a Milky Way-like galaxy in weak ( $0.01a_0$ ) and strong ( $2a_0$ ) external fields. Escape speeds in the disk plane for various field directions (solid and dotted) are compared with the error bar for the local escape speed measured from the RAVE survey (Smith et. al. 2007). The predicted circular speed curves (dot-dashed lines) are also compared with data (diamonds) (Caldwell & Ostriker 1981). Note that for an external field of  $0.01a_0$ , the transition radius between MOND and Keplerian regime is at  $\sim 500$  kpc, while it is at  $\sim 150$  kpc for an external field of  $0.03a_0$ .

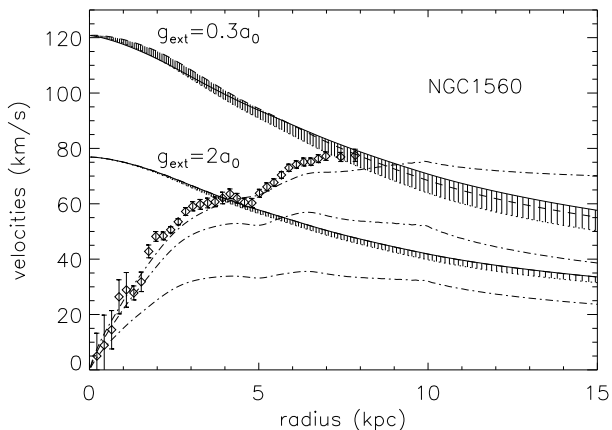


FIG. 2.— Similar to Fig.1, but for models of a NGC 1560-like LSB galaxy. Circular speed curves for  $g_{\text{ext}} = 0$  (no escape),  $g_{\text{ext}} = 0.3a_0$  (middle) and  $g_{\text{ext}} = 2a_0$  (bottom) are compared with the observed rotation curve of NGC 1560.

$[545, 558] \text{ km s}^{-1}$  in the solar neighborhood. We moreover point out that satellites of Milky Way-like galaxies will move on nearly Keplerian orbits at very large radii ( $> 100$ - $500$  kpc depending on the external field strength), with a speed much below the asymptotically flat naive MOND prediction.

We also model NGC 1560, a benchmark Low Surface Brightness (LSB) disk galaxy (Broeils 1992). We use an exponential stellar disk of  $1.97 \times 10^8 M_\odot$  and a multi-Gaussian gaseous component of  $1.07 \times 10^9 M_\odot$  to match the observed baryon distribution (Broeils 1992). A MOND Poisson solver (Ciotti, Londrillo, & Nipoti 2006) with  $256 \times 64 \times 64$  grid points is applied, the radial grid being  $r_i = 10.0 \tan \left[ (i + 0.5) \frac{0.5\pi}{256+1} \right] \text{ kpc}$ . Again it is found that an acceleration of  $0.01a_0$  is compatible with the rotation curve of NGC1560, which is an isolated galaxy.

### 3. FAST-ACCELERATING GALAXIES IN CLUSTERS

Now consider boosting the Milky Way's systematic acceleration *suddenly* to match the environment in a galaxy cluster. Fig. 1 shows for an external field of  $2a_0$ , the escape speed of stars is much reduced, falling Keplerian-like  $300\sqrt{5 \text{ kpc}/r} \text{ km s}^{-1}$  outside 5 kpc, where half of the stars and gas of the Milky Way are located. All dwarf satellites of the Milky Way, and outer disk rotating with  $200 \text{ km/s}$  would then barely be kept from flying away.

In fact, the instantaneous hypothetical circular speed must also be lowered by the sudden boost of acceleration, and the outer galaxy ( $> 5$  kpc) should exhibit a Keplerian falling rotation curve (Fig.1). Outer disk stars and gas should enter elliptical or parabolic orbits of the same angular momentum if allowed to respond to a suddenly reduced gravity, and precess with a preferred direction of instantaneous systematic acceleration which thickens the disk. In any case, observing asymptotically flat rotation curves for purely baryonic Milky Way-like galaxies residing in such environments would falsify MOND. Galaxy of same luminosity would have lower velocities, consistent with the observed trend with cluster Tully-Fisher relation (Sanders & McGaugh 2002).

Now let us consider suddenly boosting the acceleration of our benchmark LSB galaxy to  $\dot{v}_{\text{com}} = 0.3a_0$  or  $\dot{v}_{\text{com}} = 2a_0$  (typical of outer and inner parts of galaxy clusters. The real orbit of a member galaxy would pass both regions at apocenter and pericenter respectively). The circular speed (Fig. 2 lowest dot-dashed curve) of a fast-accelerating LSB is very much reduced. All previous disrupting effects are even more severe on an LSB galaxy with the escape speed falling as low as  $50 \text{ km/s}$ . Outer stars with original circular speed  $50 - 80 \text{ km/s}$  would enter parabolic orbits, and inner stars move outwards on severely elongated non-planar orbits. Actually, the dynamics resembles a purely Newtonian disk without a round stabilizing dark halo, meaning that the galaxy would become extremely bar-unstable (Mihos, McGaugh & de Blok 1997). Such an LSB would lose its MOND support and would be subject to strong distortions, even before the traditional tidal effect becomes important.

### 4. CONCLUSION AND DISCUSSION

The external field effect is a generic prediction of the modified gravity theories where the modification is acceleration-based and violates the Strong Equivalence Principle, e.g., the relativistic versions of the AQUAL theory of Bekenstein & Milgrom (1984). The effect is helpful (i) to allow high velocity stars to escape from Milky Way-like field galaxies, and (ii) to decrease the orbital velocities of their satellite galaxies at very large radii ( $100 - 500$  kpc), contrary to the naive expectation of MOND without external field effect, i.e. that rotation curves should be asymptotically flat. In this respect, the data of Klypin & Prada (2007) will be very useful in the future to verify/falsify/quantify this effect thanks to detailed numerical modelling.

On the other hand the internal dynamical structure of a field galaxy would transform when entering a cluster. Classical relations of field galaxies, such as the Tully-Fisher relation, the galaxy luminosity functions, the Hubble type distribution etc. are expected to mod-

ify strongly in clusters. The effects are most destructive for classical LSB galaxies; curiously their field counterparts have been a legendary success for MOND in terms of well-fitted rotation curves.

We thus argue that it would be extremely valuable to analyse the kinematics of a *sample* of HSB galaxies and search for LSB galaxies in nearby clusters using deep HI surveys. The study of a sample of galaxies would be needed because of the uncertainty of the determination of the real distance (as opposed to projected distance) of a galaxy from the cluster center. An obvious difficulty will be that cluster galaxies are HI-deficient (Solanes et al. 2001). An example of such an HI database is the VIVA survey (VLA Imaging of Virgo in Atomic Gas (Chung, van Gorkom et. al. 2007)). We also predict that a future detection of any undistorted HSB late-type disk galaxy near the center of a galaxy cluster would be extremely surprising in the context of MOND. A null-detection of thin LSB disks is predicted in clusters because they most probably have been turned into gas-poor dwarf ellipticals if not fully disrupted. Low surface density gas in galaxies also suffers ram pressure stripping while moving in gaseous clusters; gas is easily stripped in the reduced MONDian internal gravity, further reducing available mass for self-gravity.

Surely similar effects occur in the context of cored Dark Halos. Some simulations show that LSB disks and dwarf irregulars get harassed (Moore 1999) and transformed into dwarf ellipticals or ultra-compact dwarf ellipticals (Evstigneeva et al. 2007; Cortese et al. 2007) in the densest part of the cluster coinciding with the region where the external field is the highest. To our knowledge the properties of cluster disk galaxies (such as their Tully-Fisher relation) have not been extensively simulated. The important assumption of existing simulations is a large core for the CDM halo of the LSB; the harassment becomes much less effective if the cluster member LSB started with a dense CDM cusp density (Lucio Mayer 2007, private communications).

<sup>2</sup> For similar reasons open clusters in the solar neighbourhood are predicted internally unbound in MOND.

As for the gas-poor non-rotating dwarf spheroidals (e.g., Sextans), they are expected to have a central CDM density of  $\sim 0.1 M_{\odot} \text{pc}^{-3}$ , a factor of 100 denser than the galaxy cluster, hence might survive the tidal harassment in CDM. If in MOND a spheroidal object of  $M = 2 \times 10^5 M_{\odot}$ , and half mass radius of  $s \sim 500 \text{pc}$  is suddenly introduced into a galaxy cluster, it would have a central binding energy of only  $\sim \frac{GM}{s}(1+\Delta)^{-2/3}\mu_m^{-1} \sim (5 \text{ km s}^{-1})^2$ , much less than its initial internal random motion energy  $\frac{3 \times (10 \text{ km s}^{-1})^2}{2}$ , hence is quickly dispersed (perhaps anisotropically).<sup>2</sup> In short, any discovery of a sample of classical LSB galaxies in clusters would favor cuspy CDM, and falsify MOND or cored Dark Halos.

The tidal harassment effect exists in MOND as well (Zhao 2005); cluster galaxies suffer from tides in addition to the unique destructive effect of the external field. An even more curious distortion to the MONDian LSB or HSB disk happens when the disk is mis-aligned by an angle  $\theta_m$  with the instantaneous direction of the external field, which generally changes amplitude and direction along the orbit of an LSB on time scales of 0.2-1 Gyr. The elliptical potential of Eq.(4) creates a differential force with a component normal to the disk, hence a specific torque  $-\mathbf{r} \times \nabla \Phi$ . This causes differential precession of the disk angular momentum vector with an angular speed proportional to  $\mu_m^{-1} \sqrt{GM/r^3} \Delta \sin(2\theta_m)$ ; an LSB disk is likely shredded by one precession and a HSB disk is thickened. The *precession, asymmetric dilation and reduction of inner circular velocity curves* (cf. Fig.2) are confirmed by N-body simulations using the code of the Paris group (Tiret & Combes 2007) in MOND, but are generally forbidden by Newtonian laws in the Dark Matter framework.

We thank Luca Ciotti, Pasquale Londrillo, Carlo Nipoti for generously sharing their code, and Tom Ritchler for helpful comments on an earlier manuscript.

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